A *real* monetary business cycle?

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Abstract

Building on work by Jermann (1998) and Boldrin et al. (2001), I include a monetary sector into a standard RBC model with inelastic physical capital supply and habit persistence in consumption, which is modeled using a cash-in-advance type constraint. But in conjunction with cash, the household also self-produces credit which rises with the nominal rate of interest, so as to result in an LM-type demand for real money balances (see Kejak and Gillman, 2005; Gillman and Kejak, 2008). The demand for real money balances rises with consumption, but it also falls with the nominal rate, as proportionately more credit is produced. As a result of modeling investment (the supply of physical capital) to exhibit a quadratic adjustment cost term as in Canzoneri et al. (2007), real and nominal rates are found to *fall* following a positive shock to productivity, and the household reallocates it's liquidity portfolio in favour of money. The model is capable of exhibiting pro-cyclical endogenous demand for real money balances which also leads consumption, and the inverted indicator feature of real and nominal rates with respect to both output and consumption. It also explains procyclical and highly persistent inflation expectations and in addition provides a possible explanation for the so-called price puzzle in the structural VAR literature. In general, the model's responses to productivity innovations are observationally indistinguishable from a New Keynesian model's response following a monetary expansionary innovation on the Taylor Rule. But in the present model, real and nominal variables' responses result exclusively from productivity-driven movements of a flex-price Wicksellian real rate of interest.

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1 Introduction

Typical macroeconomic models used in policy-making during the late 70s, 80s and early 90s often combined the IS-LM framework, modeling aggregate demand, with a Phillips-type aggregate supply function and some backward-looking inflation expectations formation mechanism, dubbed the neoclassical synthesis. The *actual* inflation process emerging in such models was determined by the interplay of an exogenously specified Friedman-type monetary growth rate rule, perhaps with a bit of randomness, and an LM-type money demand function in some scaling variable related to expenditure, such as output, and in the nominal rate of interest, where this latter feature was often either motivated by some optimal portfolio allocation argument between money and bonds, or a Baumolean resource cost (or inventory cost) shoeleather story. The derivatives of this money demand function were therefore positive in the expenditure scaling variable, but negative in the opportunity cost nominal interest rate variable. Using dots to denote instantaneous changes, equilibrium in the money market was given by:

$$\dot{m} - \dot{p} = \eta_y \dot{y} - \eta_R \dot{R} \tag{1}$$

where η_y and η_R are the money demand elasticities with respect to the expenditure scaling variable and the nominal rate of interest, respectively. Like in any other market of exchange, in the money market - which is linked to the goods market - too the observed ex-post actual and ex-ante expected values of *real* money balances actually held in form of some narrowly defined monetary aggregate, the observed inflation rate, the nominal rate of interest and expenditure variable are all *endogenously* determined *simultaneously.*

A more modern-age general equilibrium framework which perhaps comes closest in replicating this argument by modeling the existence of a market for money - or for liquidity in general - explicitly, is epitomised by the cash-in-advance model (see Lucas, 1982; Svensson, 1985; Stokey and Lucas, 1987; Cooley and Hansen, 1989, 1995), in which money balances are predetermined beginning-of-period net wealth and actual money balances available for expenditure on the *consumption* good depend also on some net transfer (or lump-sum taxation) by the government, determining the growth rate of money and in standard cash-in-advance models typically also the rate of inflation (the change in the money price of the consumption good). This money market is, for the maintained assumption of strictly positive nominal rates, modeled by a strictly binding cash-in-advance constraint, in nominal terms given by:

$$M_{t-1} + T_t \equiv M_t = P_t c_t \tag{2}$$

or in real terms, given by:

$$\frac{m_{t-1}}{1+\pi_t} + \tau_t \equiv m_t = c_t \tag{3}$$

where $\tau_t = T_t/P_t$. Notice that the original cash-in-advance formulation absent of a preference-based explanation of a credit good, as in Stokey and Lucas (1987), implies a counterfactual money-consumption velocity of unity. The current-period net transfer is typically modeled such as to imply an increase of money balances at some (random) growth rate, thus leading to:

$$\tau_t = \Theta_t \frac{m_{t-1}}{1+\pi_t} \equiv (\Theta^* + e^{u_t} - 1) \frac{m_{t-1}}{1+\pi_t} \tag{4}$$

Using this, and recalling the well-known inflation tax result leading to a distortion of the marginal rate of substitution between consumption (the cash good) and leisure (see Cooley and Hansen, 1989), the cash-in-advance constraint can also be written as:

$$\frac{m_{t-1}(1+\Theta_t)}{1+\pi_t} = c_t(\tilde{z}_t, \pi_t) \tag{5}$$

where I have pointed out, that in general equilibrium endogenous consumption depends (negatively) on the inflation tax and in general also on some other variables which I have summarised here in the catch-all variable $\tilde{z}_t$. In spite of using a "catch-all-variable" here to make this point more general, it is well-known that in a two-shock (money and productivity) prototypical monetary real business cycle model as in Cooley and Hansen (1989, 1995), it is mostly the goods productivity shock, to which consumption reacts proportionately, albeit in Friedman-type consumption smoothing fashion. In such a prototypical monetary CIA business cycle model with production and physical capital, in which the representative household has a strong consumption smoothing objective, consumption varies very little [1] , so that *exogenously modeled money supply* growth rate shocks with some persistence lead to almost one-for-one adjustments in the unexpected component of inflation due to the unexpected money growth rate innovation, leaving very little adjustment in the expected inflation component along the expected transitional path back to the steady state, which is only due to modeling the money growth rate process with some persistence.

White-noise money growth rate innovations would thus, ceteris paribus, leave conditional inflation expectations over the business cycle completely unaffected following a positive innovation to money growth (see also Walsh, 2003,

[1] depending on the specification of the utility function, where in particular iso-elastic specification with high relative risk aversion makes the consumption process particularly smooth, as risk-aversion across states also implies risk-aversion across time.

chp.3). Notice also that in such models, following a, say, positive *productivity* shock without a corresponding money supply growth rate accommodation in upward direction, both actual and expected inflation typically fall below their steady state values, as pre-determined real money balances need to be adjusted upwards such as to provide the household with sufficient real money balances to buy the consumption good, whose demand has risen proportional to the productivity shock. In other words, given insufficient growth in money supply to keep up with the growth in money demand (or consumption demand), the growth rate of the money price of goods has to fall sharply below it's steady state value, in order to establish equilibrium in the money market in the period of the shock, only to grow at an increasing rate so as to converge to the steady state inflation rate from below.

Early formulations of cash-in-advance models spelled out in endowment environments (see Lucas, 1982; Svensson, 1985; Stokey and Lucas, 1987; Giovannini and Labadie, 1991) or in complete production-based real business cycle models (see Cooley and Hansen, 1989, 1995), typically proceed by modeling the growth rate of *the supply of* money balances in some *stochastic exogenously* specified fashion, using data of some narrowly defined monetary aggregate, say M0 or M1, to determine the evolution of the structural money growth rate shock. Although common practice then, in hindsight and in contrast to my earlier discussion of the LM-type money demand function I find this way of empirically estimating a model-implied *exogenously* specified money supply growth rate very puzzling.

Surely, even the most narrow definition of a monetary aggregate as *observed factually*, ought to be understood to some extent also as an endogenously determined level of money balances willingly and in some optimal portfolio-based sense held by the public, according to some well-defined *money demand* function. Indeed, it may very well be worth considering to take a complete opposite stance to the above described common practice of calibrating the *exogenous* evolution of the growth rate of the money supply *stochastically* and instead entertain the assumption of a completely *deterministic* Friedman-type growth rate rule in the *supply* of nominal money balances instead with no exogenously specified disturbances at all, and let an *endogenously* varying *money demand* function from a model explain the observed variation in real money balances held, leaving conditional inflation to vary so as to establish equilibrium between the supply of and demand for real money balances[2]. This means that one may entertain modeling the exogenously specified law of

[2] This argument is of course not novel and follows the strand of literature prevalent in the real business cycle school of thought emphasising endogenous money and reverse causation.

motion for the narrow monetary aggregate as:

$$m_t = \frac{m_{t-1}}{1+\pi_t} + \bar{v} = \frac{m_{t-1}\left(1+\Theta^*\right)}{1+\pi_t} \tag{6}$$

This is the approach I will take in this paper by formulating an endogenous-velocity production-based monetary real business cycle model, in which the representative household can self-produce a credit service, which in conjunction with money, can be used to pay for the consumption good. The specification of self-produced credit follows Kejak and Gillman (2005); Gillman and Kejak (2008) and instead of the preference-based cash-credit framework of Stokey and Lucas (1987), is a technology-based and thus Baumolean resource-based story of credit production and thus also of endogenous *money demand*, resulting from total *liquidity* demand (consumption) minus endogenous variation in credit demand (supply).

To model the exogenously specified law of motion of the supply of real balances of the narrowly defined monetary aggregate as a completely deterministic Friedman-type money growth rate rule must be understood in this context as a thought experiment to figure out how well, given this assumption, endogenous variation in the demand for real balances *alone* can produce a monetary business cycle which successfully captures the salient features of some set of real and nominal stylized facts. Given the well-known theoretical framework of the, presumably endogenously varying, money multiplier linking changes in the monetary base to the actual supply of money balances, I do not wish to convey the impression that a "rock-steady" deterministic supply of real money balances reflects reality.

More to the point is the observation that, like in any other market of exchange, *observed* quantities (money balances actually held) and prices (say, inflation and the nominal rate of interest) are merely the outcome of the interaction of some underlying theories about demand and supply of money. The purpose of this paper is to study the endogenous variation of key real and nominal quantaties and prices over the business cycle, assuming the interaction of a purely deterministic Friedman-type growth rate rule of money supply with the endogenous responses of an LM-type money demand due to productivity-driven variation in a Wicksellian real rate of interest. In contrast to this, Freeman and Kydland (2000) are using a transaction cost motivated demand for money and deposits, and given this, focus on the endogenous variation of M1 through the endogenous determination of the money multiplier.

It is exactly this last point on the interaction of money supply and demand which puts into question the common practice of specifying the *exogenous* law of motion of some money *supply* growth rate rule with the corresponding process growth rate shock by regressing some AR1 process to the *observed* growth rate series of, say, M1 and *more importantly*, by obtaining the time series properties of some exogenously specified structural money supply growth rate shock py proceeding in that particular fashion. In all likelihood, given a maintained assumption of a fairly stable evolution of the supply of a narrow monetary aggregate, what one traces out by carrying out such regressions, is actually the response of money balances held due to *money demand*, which presumably varies, in a more or less stable and predictable fashion, over the business cycle in state-contingent fashion. It is exactly this view which is taken in the present paper, which endogenizes velocity to obtain an LM-type money demand function derived from a microfounded theory of self-produced credit, which in contrast to the seminal *preference-based* cash-credit model (see Stokey and Lucas, 1987), is instead based on some *technolog-based* Baumolean resource cost view of credit production (see also Gillman and Kejak, 2008).

The model presented is of cash-in-advance real business cycle type, similar to Cooley and Hansen (1989, 1995), but the household has a *portfolio* of total liquidity supply, composed of predetermined cash plus a current transfer *and* self-produced credit, available at it's disposal to meet it's total liquidity demand (which equals consumption, as is usual[3]). Similar to Jermann (1998) and Boldrin et al. (2001), I model the supply of physical capital to be inelastic by including adjustment costs to investment. I also include habit persistence in consumption which is of internal first-difference type as in Constantinides (1990) and which has also been employed by Jermann (1998) and Boldrin et al. (2001).

Inelastic physical capital supply changes the canonical RBC model's prediction in as far as, following a positive productivity shock, interest rates *fall* as the investment boom occurs and the economy expands. Via the Fisher equation I obtain a falling nominal rate as well. Modeling the consumption process as a smoothly evolving endogenous state variable, makes total liquidity demand very smooth, which for some given state-contingent evolution of the household's liquidity supply portfolio, leads to highly persistent inflation *expectations*. The *actual* process of inflation, involving both unexpected and expected components, turns out to exhibit high-frequency variation resulting from to the unexpected component, which in turn is due to the erratic portfolio reallocation between credit and money over the business cycle. Assuming that the central bank would eliminate such high-frequency occurences of inflation variability by inelastically supplying money balances would make the highly persistent low-frequency expected inflation component matter relatively more in the determination of the actual inflation process.

Since interest rates fall as the economy expands, and credit production implies an LM-type money demand, money balances held optimally within the

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[3] I do not model investment to be carried out subject to liquidity services, as in Stockman (1981).

liquidity portfolio move endogenously procyclically over the business cycle, as observed in U.S. data (see King and Watson, 1996), and also *lead* consumption. Following a sudden fall in interest rates, the household's self-produced level of credit falls, and money demand shoots up residually, which for a given supply of money has to be accommodated by the inflation rate, which first drops sharply. In spite of the highly persistent and procyclical inflation expectations which are obtained, the initial unexpected sudden drop in inflation can be interpreted as and explanation of the robust *price puzzle* found in structural VARs, in which inflation first falls, following an expansionary innovation to monetary policy (see Sims, 1992). Simulated time series based on the goods production productivity shock alone imply a highly stable conditionally procyclical money demand over the business cycle, which by introducing direct shocks to velocity in form of credit productivity shocks, can be broken down so as to realistically mimic this apparent feature observed in post-1980 U.S. data.

All results assume a Friedman-type deterministic growth rate of the nominal balances of money supply (see Friedman, 1960), making productivity-driven responses of a state-contingent demand for real money balances function to a Wicksellian real rate of interest the main driving element in the economy. Impulse-responses from innovations in productivity lead to endogenous responses of real and nominal variables, which are generally indistinguishable from responses obtained by shocking the Taylor Rule in a prototypical New Keynesian model. What emerges then is a *real* monetary business cycle, in which productivity-driven responses in the goods market and corresponding responses in the Wicksellian real rate of interest propagate into the financial (liquidity market) in almost unidirectional fashion, with very little feedback of the financial markets back into the goods market [4] .

The remainder of the paper is organised as follows. Section 2 describes the Wicksellian banking time model consisting of a representative household, who buys consumption subject to a liquidity constraint, using a portfolio of total liquidity supply composed of predetermined money plus a current (constant) transfer and self-produced credit, and a decentralised physical capital-owning goods producing firm. The specification of exogenous shocks assumes a deterministic growth rate rule of the money supply throughout, i.e. money evolves according to some deterministic Friedman-type money growth rate rule. Following the discussion of the steady state and solution method, section 3 discusses the behaviour of the model implied by impulse responses and simulations obtained from the reduced form solution of the model. Section 4 provides a discussion, section 5 concludes.

[4] The only feedback from the financial market back to the goods market is the consumption-leisure distortion through the (expected) inflation tax.

2 The credit model

The representative agent economy is a standard monetary cash-in-advance real business cycle model (Cooley and Hansen, 1989, 1995), but is extended as in Kejak and Gillman (2005) to allow for endogenous variations in consumption velocity through the use of produced credit. The representative agent derives utility from consumption, which is of internal first-differenced habit type, as in Constantinides (1990), and leisure according to a separable utility function, which is initially specified to be iso-elastic in consumption and leisure, and given by:

$$U(c_t, c_{t-1}, l_t) = \frac{(c_t - bc_{t-1})^{1-\eta_1} - 1}{1-\eta_1} + \Psi \frac{l_t^{1-\eta_2} - 1}{1-\eta_2} \tag{7}$$

Utility therefore over the representative agent's entire lifetime (with infinite horizon) is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - bc_{t-1})^{1-\eta_1} - 1}{1-\eta_1} + \Psi \frac{l_t^{1-\eta_2} - 1}{1-\eta_2} \right], \qquad 0 < \beta < 1 \tag{8}$$

The consumer can purchase the consumption good using either money or costly produced (using banking time) credit. Denote the share of the consumption good purchased with credit $f_t^* \in [0, 1)$, then the representative household's liquidity constraint is given by[5]:

$$\frac{m_{t-1}}{1+\pi_t} + v_t \geq (1 - f_t^*)\, c_t \tag{9}$$

where $f_t^* = f_t / c_t$ is the share of total credit used in purchasing the consumption good, m_{t-1} are real beginning-of-period predetermined money balances, $1 + \pi_t$ is the inflation rate and v_t represents the governments lump-sum taxation determining the rate of money growth on the economy. Therefore, v_t satisfies:

$$v_t = \Theta_t \frac{m_{t-1}}{1+\pi_t} = (\Theta^* + e^{u_t} - 1)\, \frac{m_{t-1}}{1+\pi_t} \tag{10}$$

where Θ_t is the growth rate of money which is further decomposed into a deterministic Friedman-type constant steady-state growth rate of money Θ^* and a random component given by $e^{u_t} - 1$. Notice that randomness in the money growth rate is introduced by u_t, which follows an autoregressive process

[5] Kejak and Gillman (2005) specify the share of consumption paid for in cash directly using the variable a_t. I specify the credit share directly, instead, so that my exposition is related to their as follows: $(1 - f_t^*) = a_t$.

of order one[6]:

$$u_t = \phi_u u_{t-1} + \epsilon_{ut}, \qquad \epsilon_{ut} \sim N\left(0, \sigma^2_{\epsilon u}\right), \quad 0 < \phi_u < 1 \tag{11}$$

Credit Production is subject to a Cobb-Douglas production function, which is constant returns-to-scale in labour and consumption. This specification is motivated by the financial intermediation literature (see Sealey and Lindley, 1977; Clark, 1984; Hancock, 1985) and also by a de-centralised version of the same model in which consumption equals deposits held by a financial intermediary (see Gillman and Kejak, 2008). Credit production is therefore given by:

$$f_t = e^{v_t} A_f \left(n_{f,t}\right)^{\rho} \left(c_t\right)^{1-\rho} \tag{12}$$

While the total value of real credit is constant returns-to-scale in banking-time $n_{f,t}$ and consumption (deposits) c_t, the production of the credit share f_t^* is therefore a decreasing returns-to-scale function in consumption-normalised banking-time only, and thus given by:

$$f_t^* = \frac{f_t}{c_t} = e^{v_t} A_f \left(\frac{n_{f,t}}{c_t}\right)^{\rho} = e^{v_t} A_f \left(n^*_{f,t}\right)^{\rho} \tag{13}$$

where $n^*_{f,t} = n_{f,t}/c_t$ is the banking-time spent over total consumption (or deposits). Due to decreasing returns in the production of the credit share f_t^*, the representative household, who self-produces credit, faces an upward-sloping marginal cost curve in credit production, where f_t^* is determined by the intersection of the marginal cost curve with the opportunity cost of using the other means-of-exchange, money, which equals the (net) nominal rate of interest. Analogously to the money growth rate innovation, total factor productivity in producing credit is also random, due to v_t, which also follows an autoregressive process of order one, and is given by[7]:

$$v_t = \phi_v v_{t-1} + \epsilon_{vt}, \qquad \epsilon_{vt} \sim N\left(0, \sigma^2_{\epsilon v}\right), \quad 0 < \phi_v < 1 \tag{14}$$

Further, the representative household can spent her time endowment (which is normalised to 1) by taking leisure, by self-producing credit or by working in the de-centralised goods production firm. This means that the following time constraint needs to be obeyed at all times:

$$1 - l_t = n_{g,t} + n_{f,t} \tag{15}$$

[6] I specify theoretical randomness in money growth only for purposes of comparison with the literature and with Kejak and Gillman (2005). Throughout this paper the money growth shock will be set equal to it's steady state in simulations, meaning $\Theta_t = \Theta^* \quad \forall \quad t$.

[7] This shock is also set equal to it's steady state value in simulations, except for the section discussing the model's ability to mimic the breakdown of a stable money demand function.

The decentralised goods firm uses labour and physical capital to produce output y_t and is also assumed to own the physical capital and optimally invest from retained earnings. Additionally, I assume investing is subject to a quadratic adjustment cost function, specified as in Canzoneri et al. (2007), which is given by:

$$\zeta\left(\frac{i_t^k}{k_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t^k}{k_{t-1}} - \delta\right)^2 k_{t-1} \tag{16}$$

The decentralised firm's optimisation problem, which is solved subject to the household's discount factor, is then formulated as:

$$\begin{aligned} \max_{n_{g,t+k},k_{t+k},i_t^k} E_t \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \Bigg\{ & y_{t+k} - w_{t+k} n_{g,t+k} - i_{t+k}^k \\ & + \frac{\xi_{t+k}}{\lambda_t}\left[i_{t+k}^k - \zeta\left(\frac{i_{t+k}^k}{k_{t+k-1}}\right) - k_{t+k} + (1-\delta) k_{t+k-1} \right] \Bigg\} \end{aligned} \tag{17}$$

where the multiplier on the firm's capital accumulation constraint is equal to marginal utility in steady state only, i.e. $\bar{\lambda} = \bar{\xi}$. This means that Tobin's q is given by the ratio of $q_t = \xi_t / \lambda_t$, which is equal to one in steady state, but varies over the business cycle due to the adjustment costs and the capital gains or losses of installed physical capital. Taking first-order conditions with respect to investment, end-of-period physical capital and goods sector labour hired from the household, leads to the following conditions of optimality:

$$\lambda_t = \xi_t \left[1 - \zeta_i'\left(\frac{i_t^k}{k_{t-1}}\right) \right] \tag{18}$$

which is the first-order condition with respect to investment, where $\zeta_i'\left(\frac{i_t^k}{k_{t-1}}\right)$ is the derivative of the adjustment cost function with respect to investment. Optimality with respect to physical capital yields:

$$\xi_t = \beta E_t \left\{ \lambda_{t+1} r_{t+1}^k + \xi_{t+1} \left[(1-\delta) - \zeta_k'\left(\frac{i_{t+1}^k}{k_t}\right) \right] \right\} \tag{19}$$

Finally, the first-order condition of optimality with respect to goods production labour yields the usual condition of:

$$w_t = \alpha \frac{y_t}{n_{g,t}} \tag{20}$$

2.1 Steady State Calibration

The above table summarises the baseline calibration of the banking time model. The calibration of the model is carried out to be in line with standard

Table of benchmark calibrated Parameters			
β =0.987	discount factor	ρ =0.22	credit labour param.
α =0.64	goods labour param.	f^* =0.30	credit-to-cons ratio
η_1 =1.00	curv. param. cons.	η_2 =1.00	curv. param. leisure
A_g=1.0	TFP goods	A_f=1.461	TFP credit
l =0.7	leisure	n_f =0.00061	credit labour
Θ =1.0125	money g.	n_g =0.29939	goods labour
b =0.8	habit pers.	κ =2.0	cap. adj. cost
ϕ_u =0.70	AR money g. shock	ϕ_z =0.90	AR goods shock
ϕ_v =0.95	AR credit shock	$\sigma_{\epsilon z}$=0.0075	s.d goods shock
$\sigma_{\epsilon u}$=0.01	s.d. moneyg shock	$\sigma_{\epsilon v}$=0.01	s.d credit shock

Table 1
Baseline Calibration

values used in the literature hitherto. In particular, I calibrate the discount factor at $\beta = 0.987$, slightly below the usual 0.99 found otherwise in the literature, where my value implies an annualised steady state value of the real rate of interest of roughly 5.2%. I then chose a steady state growth rate of the money supply equal to 1.0125, resulting in an annualised steady state rate of inflation equal to 5%, where the two aforementioned calibrated values together imply an annualised steady state value of the nominal rate of interest equal to 10.2%. Calibrated values for the goods production sector are standard. I choose a labour share in production equal to $\alpha = 0.64$, steady state total factor productivity in the goods sector to be normalised at $A_g = 1.0$ and the steady state amount of leisure to be $l = 0.7$, implying a steady state of *total* labour residually equal to 0.3, which has to be allocated between goods and credit production according to the labour market equilibrium condition of equating marginal (revenue) products of labour:

$$w = \alpha \frac{y}{n_g} = i\rho \frac{f}{n_g} \tag{21}$$

I calibrate the steady state share of credit production $f^* = 0.3$, which follows Kejak and Gillman (2005) and roughly matches the observed steady state long-run behaviour of consumption velocity in U.S. data. Also, following Kejak and Gillman (2005) who obtain their value of the labour share in credit production equal to 0.21 from an empirical time series study conducted by Gillman and Otto (2005), I calibrated this value at $\rho = 0.22$, which is only slightly above theirs. Given the above set of calibrated parameters, I use a nonlinear equation solver and residually obtain $A_f = 1.461$ and $n_f = 0.00061$, which is very close to the corresponding values of 1.422 and 0.00049 obtained by Kejak and Gillman (2005). The calibration of the habit persistence parameter in consumption is as in Constantinides (1990) and fixed at $b = 0.8$, whereas I calibrate the investment adjustment cost parameter $\kappa = 2.0$, which is only one-quarter of the calibration chosen in Canzoneri et al. (2007), so as to model only a small amount of frictions regarding the supply of physical capital. The shock processes in the model are also specified along standard values. In particular I model the persistence in the goods sector total factor productivity shock to be equal to $\phi_z = 0.90$, which implies slightly less persistence than the usual chosen value of 0.95, with a standard deviation of the iid innovations equal to $\sigma_{\epsilon z} = 0.0075$ (as compared to 0.00721 Cooley and Hansen (1989)). The autoregressive parameters for the money supply growth rate shock u_t and the credit total factor productivity shocks v_t are 0.6 and 0.95, respectively, where the former choice follows Kejak and Gillman (2005) and the latter follows the common calibration for productivity persistence found in the literature. The corresponding standard deviations for the iid shock innovations are calibrated at $\sigma_{\epsilon v} = 0.01$ and $\sigma_{\epsilon v} = 0.015$. However, I want to emphasise at this point, that the chosen values describing the evolution of the money growth rate shock are completely irrelevant for the present study, as throughout the paper u_t will *never* receive any shocks. Notice also that I do actually include the exogenous law of motion of the money supply growth rate shock into the state of the model, but here since shocks are set to zero, even the calibrated autoregressive parameter ϕ_u does not play any role, as the first-order conditions of the model never involve any forward-looking prediction of $E_t m_{t+1}$ or $E_t u_{t+1}$. Also, for the majority of my discussion, the exogenously specified evolution of the credit productivity shock v_t is also assumed to receive no shocks in form of iid innovations on it's law of motion, except for a discussion towards the end, in which I discuss the model's ability to replicate the breaking down of a stable money demand function. Again, I actually do include the law of motion of v_t when solving the model, but since it never gets shocked and no forecasts of $E_t f_{t+1}$ or $E_t v_{t+1}$ are involved, the specification of the autoregressive root ϕ_v does not matter.

2.2 Solution & Competitive Equilibrium

The model's first-order conditions of optimality, market equilibrium identities and non-linear specification of the shock processes are linearised by taking a first-order Taylor series expansion around the (log of the) steady state. I obtain a linear system of expectational difference equations given by:

$$\mathbf{A}E_t \begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_t \\ \mathbf{y}_{t+1} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \end{bmatrix} \tag{22}$$

where $\mathbf{z}_t = [z_t, u_t, v_t]'$ is a vector containing the current-period exogenous structural shocks, $\mathbf{y}_t = \left[w_t, \pi_t, y_t, l_t, n_{g,t}, n_{f,t}, i_t^k, \lambda_t, \mu_t, \xi_t, f_t, R_t, I_t, q_t\right]$ is a vector containing the current-period endogenous control, or "jump" variables and $\mathbf{x}_{t-1} = [m_{t-1}, k_{t-1}, c_{t-1}]$ is a vector containing the endogenous states of the system. All variables collected in vectors are now log deviations from steady state. Since I have not reduced the system by hand, as is done in King et al. (1988) and also shown in Uhlig (1995), so as to eliminate intra-temporal equations, the matrix $\mathbf{A}$ is generally going to be singular, disallowing direct use of the BK diagonalization technique (see Blanchard and Kahn, 1980). So instead I use the triangularization technique developed in Klein (2000) to solve for the reduced-form solution of the system which is given by the recursive law of motion of the system:

$$\left[\mathbf{y}_t\right] = \mathbf{F}\begin{bmatrix}\mathbf{z}_t \\ \mathbf{x}_{t-1}\end{bmatrix} \tag{23}$$

and for the evolution of the state of the system:

$$\begin{bmatrix}\mathbf{z}_{t+1} \\ \mathbf{x}_t\end{bmatrix} = \mathbf{P}\begin{bmatrix}\mathbf{z}_t \\ \mathbf{x}_{t-1}\end{bmatrix} + \begin{bmatrix}\epsilon_{t+1} \\ \mathbf{0}\end{bmatrix} \tag{24}$$

3 Results

In this section I am going to analyse and discuss simulated evidence from the artificial banking time model. To this end, the reduced-form solution of the linearised rational expectations model (i.e. the recursive law of motion) is used to produce impulse responses and simulated time series. Since the model is a prototypical monetary cash-in-advance RBC model, as discussed in Cooley and Hansen (1995), most of my attention will be focused on the economy's response to an innovation in the goods sector total factor productivity parameter.

For most of my simulation results, I will first consider holding the credit production productivity shock fixed at it's steady state value, so as to dissallow direct production-based shocks to velocity measures implied by this shock affecting the position of the marginal cost curve in credit production. In order to better capture the historically observed disassociation of narrow monetary aggregates from the business cycle in post-1980s data of the U.S. and U.K. and other countries, in a separate simulation exercise I will consider this direct shock again.

Figure (1) illustrates the economy's response to a 1% innovation in the goods

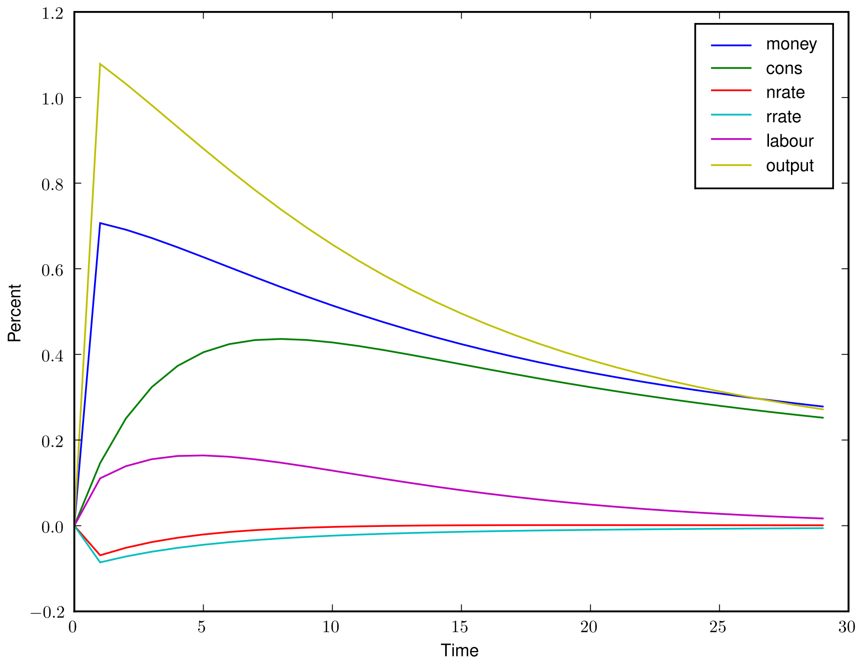

Fig. 1. 1% innovation in productivity

sector total factor productivity. Many of the observed responses are well-known from the canonical RBC framework, such as a Hansen RBC model with divisible labour Hansen (1985). However, three features of the present *monetary* RBC model stand out and beg explaining. In contrast to a standard RBC model (which does not contain a monetary sector and is thus incapable of being informative about nominal variables), following a shock to productivity, both the real and nominal variables fall and real money balances held rise. Further more, the real rate falls by more than the nominal rate, implying a rise in inflation *expectations*, through the Fisher equation.

Incidentally, this latter set of effects is also a typical property of models of the new neoclassical synthesis school, in which the shock producing such responses is however assumed to affect nominal rates directly through some innovation to a Taylor Rule implying an unexpected shock to monetary policy (a monetary expansion). Therefore, the present model's response to a *productivity* shock bears a striking resemblance to the response obtained from a prototypical NNS model receiving an innovation to the Taylor Rule. However, whereas the latter model implies a direct nominal interest rate control exerted by central banks, the former model produces a productivity-driven Wicksellian story of the natural real rate of interest, which for some endogenous response in inflation and inflation expectations, implies a corresponding Wicksellian story for the nominal rate as well.

The model's key building blocks, based on habit persistence in consumption and some degree of adjustment cost to investment, essentially results in an economic environment similar to Jermann (1998) and Boldrin et al. (2001), both of which are studies emphasising the explanation of asset pricing regularities. Boldrin et al. however, discuss how their *real* business cycle model (which does not contain a monetary sector) can account surprisingly well for various business cycle facts and represents an improvement over the canonical RBC model. Boldrin et al. also discuss how the improved picture results from a combination of habit persistence, inelastic physical capital supply and input factor market rigidities (i.e. some degree of labour market immobility as well).

An important element missing in their story disallowing a *direct* comparison with NNS models based on nominal rigidities is the incorporation of a story for money demand so as to be able to model nominal variables as well. The present study fills this gap through the inclusion of a cash-in-advance type constraint and interest-elastic credit production, globally implying a Cagan-type money demand function with a non-linearly *falling* interest-rate elasticity, but around some local steady state (i.e. some fixed calibrated nominal rate of interest) a simple LM-type money demand function with a *fixed* interest rate elasticity. Before elaborating more fully on the underlying mechanisms leading to the observed impulse response of the present model to an innovation in productivity, a discussion of the behaviour of nominal variables following that shock will be carried out next.

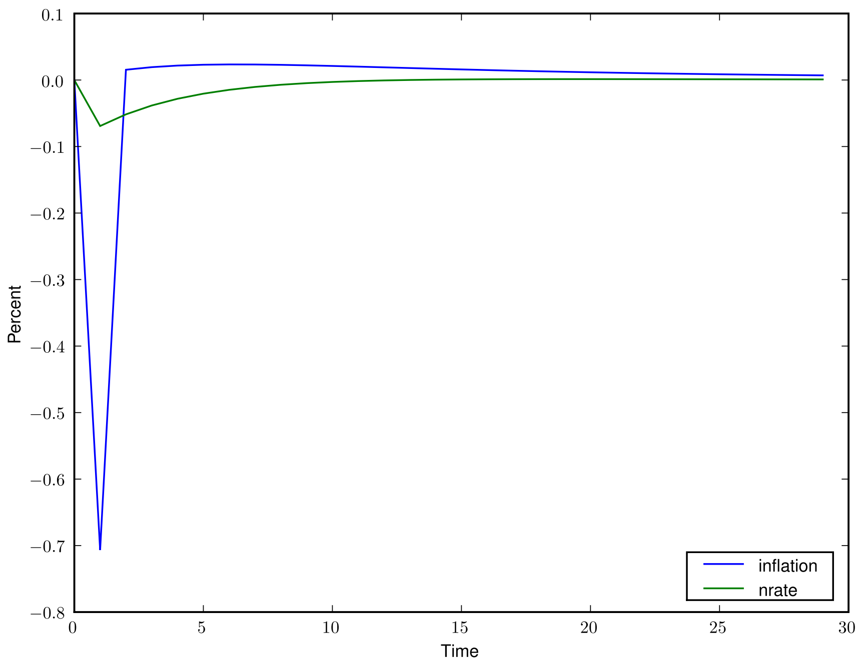

Fig. 2. 1% innovation in productivity

Figure (2) illustrates the banking time model's response of the nominal rate of interest and the inflation rate to an innovation in productivity. A standard (monetary) RBC model will typically exhibit a *rise* in both real and nominal rates following a shock to productivity, followed by an investment boom. The investment boom occurs here as well, but it coincides with a fall in both rates. This has also been observed by Boldrin et al. (2001). I will defer a discussion of this particular detail until later, when a full narrative of the model's mechanism will be presented. Focusing on the inflation response, figure (2) reveals a sudden unexpected drop in the inflation rate. Given a fixed Friedman-type growth rate of the money *supply*, a sudden drop in the nominal rate implies also a sudden drop in the level of self-produced credit, thus residually resulting in a sudden increased demand for real money balances. For a given fixed money supply growth rate, the sudden increased growth in the demand for real money balances has to be accommodated by an unexpected fall in the rate of inflation below it's steady state value, so as to adjust the *supply* of beginning-of-period predetermined real money balances upwards to be in line with real money demand again.

Notice that in all of my discussions in relation to liquidity market responses following a shock to productivity, throughout I will assume a strict Friedman-type money supply growth rate rule, i.e. for now I will refrain from giving money supply any meaningful policy-motivated purpose, which it may however have in the usual state-contingent fashion. Having said that, although this sharp *unexpected* drop in inflation below it's steady state value appears drastic, clearly a central bank would rush to dampen such drastic drops by inelastically supplying sufficient money (through open market operations) such as to counteract any looming high-frequency volatility in the rate of inflation (of course, in practice this is done at the level of the high-powered monetary base, and not to dampen high-frequency volatility in inflation, but rather in the nominal rate). In as far as as such a response, though partially succeeding in dampening sudden inflationary swings, may still occur too late, it would still lead to an initial fall of the rate of inflation following a shock to productivity and an expansion of the economy. The negative response of *actual* inflation in the banking time model could therefore possibly be interpreted as an explanation of the so-called *price puzzle*, which has been found in analyses of impulse-responses obtained from identified VARs (see Sims, 1992; Eichenbaum, 1992).

Another perspective on this is to realise that the model provides the representative household only with a very simplified liquidity portfolio, in which nominal interest rate movements lead to strong substitution effects from credit to money and vice-versa. In the real world, other means of exchange - perhaps other production-based credit-like or indeed fiat-based money-like exchange services with varying interest rate elasticities - would of course lead to a more diversified liquidity supply portfolio and thus to different money demand responses to changes in the nominal rate of interest. Alternatively, focusing on the production- (or technology-) based property of self-produced credit, a "time-to-built" specification may be introduced such as to make the expansion of credit more sluggish.

However, having touched upon such possibilities, given empirical evidence of the U.S. (see King and Watson, 1996) on the largely procyclical and *leading* role of narrow monetary aggregates with respect to output and consumption, such evidence may be indicative of the fact that - following shocks to productivity and immediate *real* Wicksellian changes of interest rates - the reallocation of means-of-exchange within a broader portfolio of liquidity supply in response to this appears to occur in contemporaneous fashion, whereas goods markets respond late. This evidence is compatible with the view that financial markets react fairly quickly to Wicksellian (productivity-driven) interest rate changes, thus putting into question some of the above entertained modifications to introduce some degree of sluggishness into the liquidity market. In the model presented, goods markets lag in consumption, clearly due to habit persistence, but may also - so far absent in this model - generally lag in output as well, either by modeling labour as less mobile, or perhaps, as in Cochrane (1988, 1993); Belo (2007); Jermann (2006), by allowing the firm to intertemporally smooth the arrival of an underlying productivity shock into the *effective* productivity shock based on the current period's valuation implied by the household's discount factor.

The important point here is that in the model presented, liquidity markets, both in quantities and prices, react contemporaneously to productivity-driven movements of a Wicksellian real rate of interest (and via the Fisher equation, corresponding movements of the nominal rate as well), whereas goods markets respond slightly sluggishly because of habit persistence in consumption. Notice that although output moves essentially contemporaneously with the productivity shock, as labour market real rigidities are absent, labour still partially expands in hump-shaped fashion, in as far as the maximum response of labour occurs 2 quarters after the shock. Given a humped-shaped consumption pattern and persistence in the productivity shock, labour first optimally jumps up discretely, but then also exhibits a hump-shaped segment of further smooth expansion.

It is also important to note at this point that the above obtained result contradicts Boldrin et al. in their view that the general equilibrium investment adjustment cost framework in Jermann (1998) *always* produces counterfactual *counter-cyclical* labour. I have found this to be sensitive to calibration. In particular, modeling physical capital supply *too* inelastically and/or the representative agent too risk-averse can easily lead to counter-cyclical labour. None of the models sluggish goods market quantity (consumption and labour) responses are due to sluggish price and/or wage responses and although output rises contemporaneously, in principle labour markets could also be modeled sluggishly so as to permit a more hump-shaped response in output too (see Boldrin et al., 2001). But what about the *expected transitional* response of inflation following the first-period unexpected sharp drop due to the reallocation of the household's portfolio of liquidity supply? It is this issue I will turn to next.

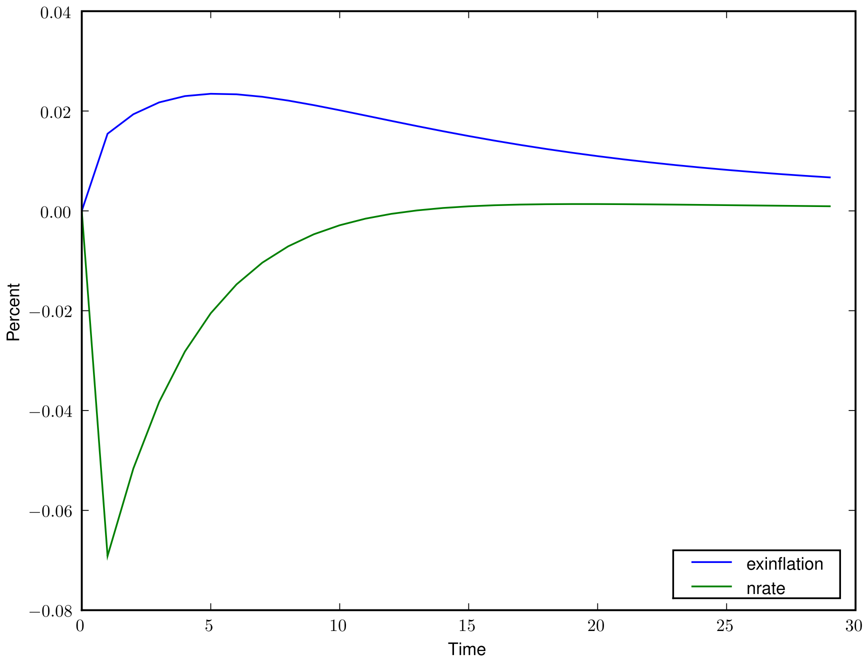

Fig. 3. 1% innovation in productivity

Following the described unexpected first-period sharp response in inflation, equating liquidity demand (consumption) and the erratic portfolio reallocation of liquidity supply (credit and real money balances) in that period, the expected transitional path of inflation back to steady state follows a pattern illustrated in figure (3). As interest rates (both real and nominal) begin to rise again, self-produced credit slowly picks up as well and real money balances demanded therefore falls slowly residually through the credit production-implied LM-type money demand function. Since along the expectational path, real money balances demanded adjust back to their steady state value from above, for a given deterministic steady state Friedman-type growth rate rule of money supply, the adjustment of the aforementioned has to come from above-steady state inflation rate which are gradually falling, so as to match money supply with the endogenously smoothly falling demand in real money balances.

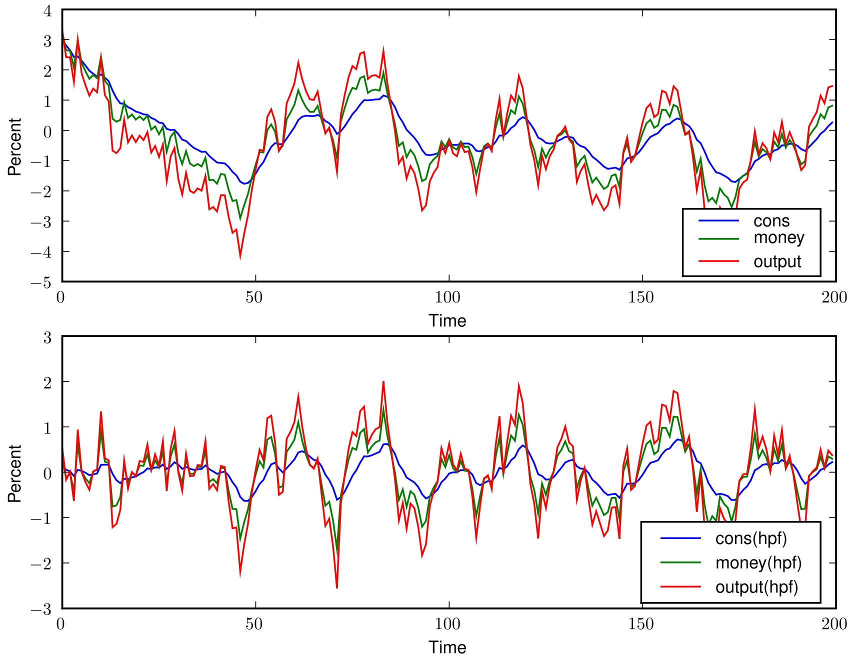

Fig. 4. Simulation Evidence: Productivity Shocks

3.2 A real monetary business cycle: A narrative

This section is going to provide a narrative of the mechanisms at work in the banking time economy, following a shock to the innovation in productivity. This will be conducted in an informal style, through which I wish to make a convincing argument for the relatively successful way the model can model the observed real-world facts. What I hope to convey here is that the model provides a Wicksellian banking time story, in which key real and nominal variables move in intuitive fashion over the business cycle and in which productivity-driven changes in the goods market affect the financial (or liquidity) market in almost uni-directional fashion, with very little feedback of the financial markets back to the goods market. So what is the *real* story behind the Wicksellian banking time model's monetary business cycle?

Following the productivity shock, the sun starts to shine and through the persistence of the Solow residual is projected to carry on shining in expectation, albeit at some slowly decaying rate. In the canonical real business cycle model, output, labour and consumption all rise proportionately with the productivity shock, but the latter series less strongly as the representative agent wishes to smooth his marginal valuation of wealth and consume in Friedman-type permanent-income implied fashion. More importantly, the standard RBC model predicts a counterfactual *rise* in real rates as the investment boom occurs and the economy expands. Why then, does the presently discussed model predict a *fall* in real rates instead? This phenomenon is directly related to the assumption of adjustment costs to investment and is also discussed in Boldrin et al. (2001). After the shock, the household wishes to transfer a lot of the "good times" into future periods, by moving resources into the physical capital storage technology, or in plain words by trying to save via the capital markets. With an inelastically modeled supply of the same, the household quickly drives up Tobin's q leading to capital gains of installed physical capital, a bull market ensues and stock markets rally! This is clearly shown in the last simulated series of figure (6), in which Tobin's q moves procyclically with output and countercyclical with real and nominal rates.

But now, through the household's enormous desire to save - which is even enhanced because of the even smoother optimal consumption pattern implied by habit persistence - it quickly becomes too costly to keep on buying capital and instead the household just has to accept a rather suboptimal[8] implementation of his consumption projection through time, meaning that he will "overeat" today relative to tomorrow and carry on doing so even in subsequent periods. This means that the marginal valuation of wealth will always be lower in any period relative to the following period, which through the first-order condition for real bonds implies a fall in the real rate below it's steady state value and a gradual rise back to the steady state from below. Notice that in principle the household could try and dampen the initial fall in the marginal valuation of wealth, $\hat{\lambda}_t$, simply by working less and taking more leisure.

But this does not occur here, because through habit persistence the household's projected (or future expected) appetite is of hump-shaped nature and is thus projected to expand with a very high root - more than 0.9 for the solved reduced form's autoregressive coefficient of consumption on it's past value. So if the representative agent were to take leisure now in the initial periods following the shock, it may dampen the fall in $\hat{\lambda}_t$ now, but it would come at the expense of not having enough physical capital in future periods, acquired through today's wage bill, to satisfy it's projected future increasing appetite. Notice that the procyclical *rise* in labour is very sensitive to calibration; modeling physical capital supply *too* inelastically by raising κ will lead to countercyclical labour in the initial periods following the shock, which would then eventually rise above steady state, but only much later, certainly too late to account for the stylized facts which clearly show labour to be procyclical (see King and Watson, 1996).

For reasons already discussed in the section on interest rates and liquidity portfolio reallocation, the sudden drop in the real and nominal rate (due to

[8] *suboptimal* relative to a standard RBC model's consumption response. Of course, given the increased cost of physical capital, the consumption response is optimal.

inelastic capital, the stock market rally and "suboptimally"[9] overeating today relative to tomorrow compared to perfectly elastic capital supply in the standard RBC model), cause a sudden drop in self-produced credit and given current (and future projected hump-shaped increases of) total liquidity demand due to consumption, demand for real money balances jumps up. For a given deterministic Friedman steady state growth rate of money supply, for a much higher one-off rise in real money demand, the inflation rate has to - unexpectedly - drop sharply below steady state to adjust money supply sufficiently. But along the expected projected path, inflation has to converge from above steady state, as gradually rising real and nominal rates imply a shift away from money and back into self-produced credit, so that money balances have to be gradually adjusted downwards again through inflation rates above steady state.

The picture that emerges is one of procyclical *endogenously* determined demand for real money balances, since inelastic capital markets lead to a *fall* in real and nominal rates following the shock to productivity. Self-produced credit and it's comparatively high interest elasticity lead to a - in terms of local dynamics around some steady state - LM-type money demand function for real money balances along implying sudden shifts in the portfolio of liquidity between credit and money. Figure (4) shows a simulation of real money balances, consumption and output over the business cycle, where the top graphs is unfiltered and the bottom is hp-filtered to remove high-frequency components. What the simulation clearly shows is that in a world of a stable LM-type money demand function, *endogenously* determined real money balances move closely together with consumption and output, and also lead consumption, because of the latter's slow response due to habit persistence. Including labour market rigidities, as in Boldrin et al. (2001) or using some other mechanism could also lead to a slower response of output, thus making endogenous money demand also lead output.

But here money neither causes output nor consumption! Following the productivity shock and a sudden fall in the Wicksellian real rate of interest, the representative household's liquidity portfolio experiences a sudden reallocation from credit to money, financial markets move fast! But the goods market's response may generally lag the same sudden drop in the productivity shock-induced real rate, here however only in consumption which exhibits habit persistence. Further below, I will also discuss and show in simulations, how the model is in principle capable of modeling the well-documented breakdown of the tight relationship between real money demand and both consumption and output, by introducing or "switching back on" the direct credit production productivity shock v_t, so as to model direct shocks to velocity.

[9] Given adjustments costs to investment, the chosen consumption profile is of course optimal. In absence of the same costs, a smoother consumption process were chosen.

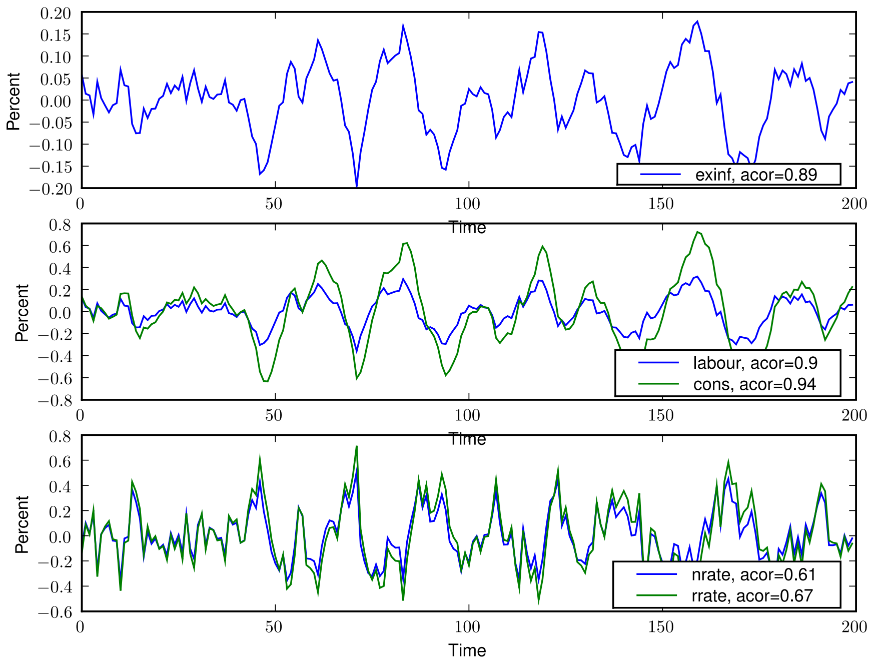

Fig. 5. Simulation Evidence: Productivity Shocks

There are also other feature revealed by simulations deserving of mention. First of all, the model is capable of exhibiting highly procyclical and very persistent inflation expectations, which are even slightly lagging expansions in output. Also, interest rates are inverted indicators and real rates move by more than nominal rates following a productivity shock. Such responses of real and nominal variables have often been associated with an innovation to *monetary policy* as implied by a negative innovation on a Taylor nominal interest rate rule. Here however, all real and nominal results are driven by a productivity-driven fall in the Wicksellian real rate of interest; the causation from goods to financial (or liquidity) markets simply happens to be such as to make the overall picture which emerges observationally equivalent to one which would follow a Taylor rule-type expansionary monetary policy innovation in New Keynesian models incorporating price and wage rigidities.

3.3 The breakdown of the stable money demand function

This section is going to demonstrate, how the discussed banking time model is in principle capable of mimicking the well-known breakdown of a fairly close association between real money balances and both output and consumption observed in U.S. data (see King and Watson, 1996). Thus far, simulation and impulse response evidence has only focused on telling a "real" story of

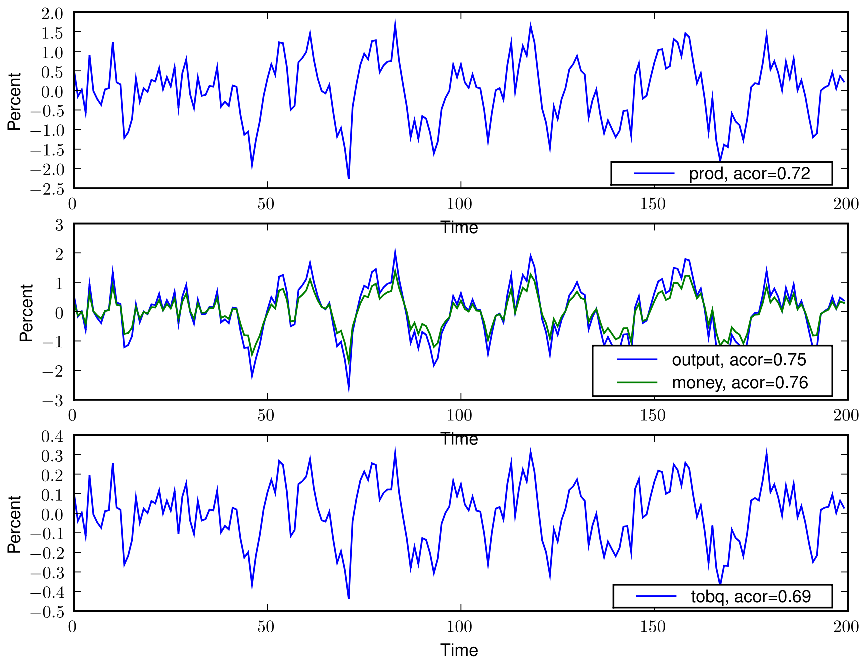

Fig. 6. Simulation Evidence: Productivity Shocks

the monetary business cycle, in as far as only shocks to goods productivity were considered. Combined with habit persistence in consumption and - more crucially - adjustment costs to investment, the model showed some success in mimicking the salient features of the U.S. monetary business cycle. Inverted indicator interest rates led to liquidity portfolio reallocation in favour of real money balances over the business cycle (i.e. making real money procyclical). As a results of sudden liquidity portfolio reallocation due to sudden changes in the Wicksellian real rate of interest, a price puzzle ensued (a sharp drop in actual inflation below it's steady state value), followed by a procyclical and highly persistent convergence of expected inflation from above it's steady state value. All results were obtained by assuming a deterministic Friedman-type constant growth rate of the nominal money supply. Countercyclical interest rates and a well-defined LM-type demand for real money balances (due to credit production's *positive* interest rate elasticity) led to the latter's intuitive behaviour over the business cycle as implied by the model. Money, output an consumption were shown to exhibit a very close contemporaneous relationship. So how then, could the model also explain the breakdown of this relationship which is such a striking feature of post-1980's data?

The answer to this question lies with the shock to credit production total factor productivity, v_t, which could be thought of as a direct shock to velocity, as it disturbs or re-positions the upward-sloping marginal cost curve in credit production, leading to changes in the share of credit over consumption that

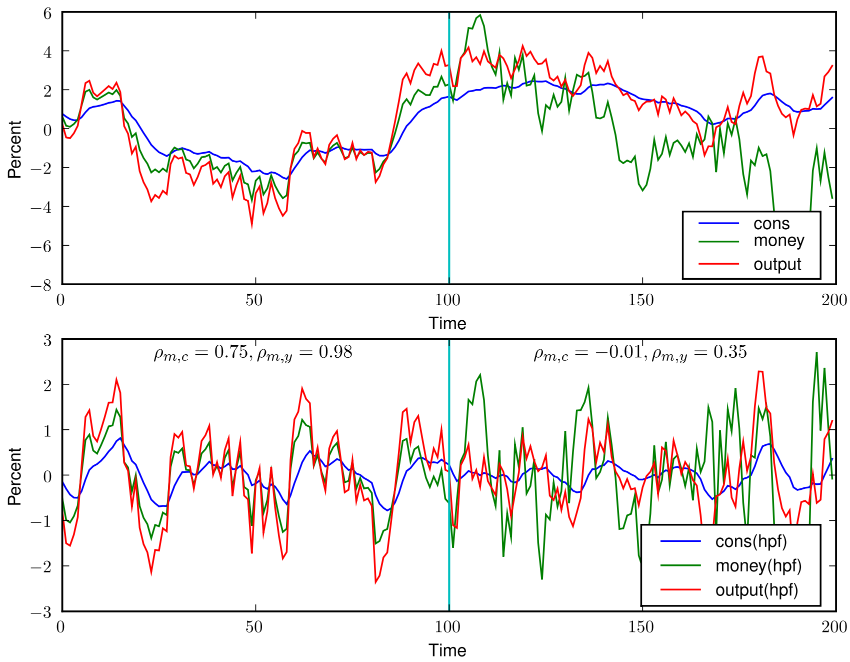

Fig. 7. Simulation Evidence: Productivity Shocks & Credit Shocks

way for any given price (i.e. the nominal rate) of credit. Figure (7) illustrates a simulation of the model, which contains exactly the same shocks to productivity as the ones used in producing figure (4) but only up to time period 100. As an illustrative exercise, beginning from period 101 onwards[10], I have fed also credit production shocks into the exogenous-endogenous state system described by:

$$\begin{bmatrix} z_{t+1} \\ u_{t+1} \\ v_{t+1} \\ m_t \\ k_t \\ c_t \end{bmatrix} = \mathbf{P} \begin{bmatrix} z_t \\ u_t \\ v_t \\ m_{t-1} \\ k_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{z,t} \\ \epsilon_{u,t} \\ \epsilon_{v,t} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

where $\mathbf{P}$ is the 6×6 matrix describing the evolution of the state of the system, which also demonstrates how habit persistence in consumption effectively turns the latter also into an endogenous state variable, instead of it assuming

[10] Strictly speaking, if one treated the simulated sample as a model-implied counterpart to the real data, one would only want to feed in credit shocks in the last quarter of that sample. For illustrative purposes I choose a 50/50 divide.

the nature of an endogenous "jump" or control variable, which it typically does in the canonical real business cycle framework. Notice that, as has been assumed throughout, innovations to the money supply growth rate $\epsilon_{u,t}$ have been set to zero during the simulations, so as to mimic a deterministic constant Friedman-type money supply growth rate rule.

The graph also reports correlation coefficients of money with consumption and output in simulated periods $0-100$ on the one hand, and the same coefficients of linear association for periods $101-200$ in which the credit shock has been added. It is thus clear to see, how the tight correlation which exists in the simulated sub-period without the credit shock, which results from "riding up and down" a stable LM-type money demand function over the business cycle, breaks down or is less strong in the simulated sub-period with credit shocks. Notice that the nature of such shocks which affect the endogenously determined technology-based liquidity portfolio composition between money balances and credit can be thought of as stemming from financial deregulation, an interpretation which has also been chosen and studied elsewhere (see Kejak and Gillman, 2005; Gillman and Kejak, 2007).

4 Discussion

Before concluding, the following section will provide a discussion of the model presented and the results which have been obtained from it. In particular, since the model presents an attempt to explain monetary business cycle facts using a *real* story based on productivity-driven changes in the Wicksellian real rate of interest, the lessons drawn from such an experiment clearly call for a comparison with the current *theoretical* consensus embodied by new neoclassical synthesis models and the conduct of monetary policy using Taylor Rule type interest rate-setting behaviour and *practical* consensus embodied by inflation-targeting.

4.1 Taylor Rules vs. Money Supply Rules

The current consensus of theory-informed monetary policy is embodied by the prescriptions emanating from models of the so-called new neoclassical synthesis, or New Keynesian school of thought emphasising in particular the existence of nominal rigidities in the goods and/or labour markets, operationalized theoretically through the incorporation of so-called Calvo contracts (see Calvo, 1983). As monopolistic intermediate goods firms are only allowed to adjust their prices in each period with some fixed probability, they have to base their current-period optimal price-setting decision using some forecast of future inflation, which will also affect their future nominal marginal cost. Thus, the purely forward-looking micro-founded Phillips curve emerges [11] .

Due to the assumption of price stickiness *and* the additional assumption that those firms which cannot adjust their prices in any given period index their prices to past inflation, such models are also capable of exhibiting a very persistent inflation process, which is therefore "built-in institutionally", so to speak (see inter alia Yun, 1996; McCallum and Nelson, 1999; Lawrence J. Christiano and Evans, 2005; Canzoneri et al., 2007; Smets and Wouters, 2007). With price-indexation, a Phillips curve incorporating both backward- and forward-looking inflation emerges. Also, a short-run trade-off emerges again between inflation and output, which in this microfounded utility maximisation-based framework does not lead to a political-economy argument of opportunistic exploitability, as this would reduce welfare of the representative household.

In such a framework, optimal monetary policy is typically specified by a Taylor Rule nominal interest rate setting description, which - if operated optimally and given the sluggish inflation process - has to track the flex-price Wicksellian real rate of interest as closely as possible, so as to minimise welfare-harming distortions in the goods and labour market. In other words, the current view is such as to describe *current-period* inflation dynamics, in some sense, to be under little control by policy makers and indeed, given the myriad of shocks hitting an economy at all times, to almost take on a life of it's own due to it's institutionalised nature given to it through the Calvo price setting framework coupled with price indexation.

So given sluggish inflation, and perfect control over the nominal rate as the main policy instrument, the central bank can control the real rate of interest as it sees fit, but perhaps would optimally want to do so in a way so as to closely track the flex-price Wicksellian rate of interest. Feasible interest rate rules, in an operational sense, as discussed for instance in Canzoneri et al. (2007), vary for instance in their definition of how to define the output gap (current minus steady state vs. current minus flex-price output) in a Taylor Rule, but may perhaps target other variables alltogether, such as nominal wage inflation (see also Canzoneri et al., 2007).

The Wicksellian banking time model, which I have described and analysed in this paper tells a different story of the world. In particular, the real rate of interest is an equilibrium price which ensues as a result of productivity shocks in the goods market, a "bull market" embodied in capital gains due

[11] Which, among other, has the unfortunate feature that announced and fully credibly anticipated monetary expansions cause recessions, as firms increase their prices ahead of time (see Mankiw, 2001). Also, Calvo price setting means that with very low probability some firm(s) may *never* be able to change their price!

to increases in Tobin's q, and the household's "overeating" today relative to tomorrow due to the increased cost of transferring wealth into future periods via the capital markets. "Overeating" today relative to tomorrow means a lower marginal valuation of wealth today versus tomorrow, which leads to a fall in the household's stochastic discount factor as implied by it's first-order condition with respect to real bonds. There are no nominal rigidities, but a procyclical and highly persistent inflation process develops nevertheless, simply due the the persistence in liquidity demand embodied by modeling consumption to exhibit habit persistence.

Since the money supply has been assumed to be of a Friedman constant growth rate type throughout, and most variation in nominal variables has been associated with the evolution of total liquidity demand (consumption) on the one hand and endogenous velocity (credit production) on the other (both of which were productivity-driven through the Wicksellian rate of interest), it would of course be natural to ask at this point whether money supply could be given some optimal state-contingent, and operationally feasible specification. Linked to this consideration is the question of what criterion the central bank would want to target in endogenously varying the money supply growth rate rule in feedback fashion. Also related to this is the usual questionable assumption of the central bank's *direct* control over the narrow monetary aggregate, or whether the model should not be amended to include some specification of the evolution of monetary base so as to link this to M1 via a general equilibrium formulation of the money multiplier, provided by the financial accelerator framework developed by Bernanke et al. (1999).

The answer to the criterion question would probably point to reducing the margin-distorting effect of the inflation-tax between consumption and leisure, whereas the question of an *operationally feasible* money supply feedback function would probably point to the late Milton Friedman's famous argument related to the "long and variable lags of the effects of monetary policy" Friedman (1961). However, in the current model the problem would not lie in the long and variable lags with which monetary policy may affect the state of the economy, but conversely in the long and variable lags with which money supply changes, embodied by an *operationally feasible* money supply feedback function, would *react* to the current state of the economy. The question of an optimal state-contingent money supply response is beyond the scope of this paper, but endogenous money supply functions in similar flex-price models have already been examined (see Gavin and Kydland, 1999).

4.2 Cash-in-advance, Habit Persistence and Real Indeterminacy

It is interesting to note at this point, that it has been shown that combining *standard production-based cash-in-advance* models as in Cooley and Hansen (1989, 1995) with habit persistence can easily lead to real indeterminacy, thus such models have no stable saddle-path solutions (see Auray et al., 2005). However, the same authors show, that real indeterminacy disappears completely when the household has access to a second means-of-exchange - allowing him to escape the cash-part of the liquidity constraint that way - which is *costless in a net wealth sense*. In particular, Auray et al. show that by endogenizing velocity using a transactions cost technology as in Marshall (1992) or Carlstrom and Fuerst (2001), does not eliminate the problem of real indeterminacy, as the consolidated real resource constraint of the household still contains the net cost embodied by the transaction cost. The present model's way of endogenizing consumption velocity is accomplished through self-produced credit, which is therefore costless in a net wealth sense and thus provides a framework in which real indeterminacy as described in Auray et al. (2005) does not pose a problem.

4.3 The Price Puzzle in identified SVARs

In some sense, complementary to the analysis of *calibrated* artificial economies by means of studying simulations and impulse-responses obtained from reduced-form solutions of approximated non-linear rational expectations systems, is the structural VAR (sVAR) literature, which instead is based on *estimation* of the coefficients of unrestricted VARs using data and some *non-unique* orthogonalization method to identify the "true" structural shocks (see Sims, 1986; Bernanke, 1986; Blanchard and Quah, 1989) [12] . This approach has been employed in particular to empirically evaluate the importance of monetary policy shocks in the determination of endogenous response of business cycle quantities and prices (see Leeper et al., 1996; Christiano et al., 1999; Uhlig, 2005) and, due to the non-uniqueness of identification, is not without it's critics (see Rudebusch, 1998; Chari et al., 2005).

One part of this literature specifically concerning itself with the analysis of the importance of monetary policy shocks (and thus concerning itself empirically with the monetary transmission mechanism) based on impulse responses, has led to the identification of a very robust phenomenon, called *the price puzzle* (see Sims, 1992). Contrary to many economists' a-priori expectations about

[12] Many different orthogonalization schemes have been employed, ranging from the "standard" Cholesky decomposition, to schemes based on long-run restrictions.

the response of the economy to a, say, positive shock to monetary policy, impulse responses from estimated and identified VARs show an initial *fall* in the price level, following the positive innovation to the short-term nominal rate [13] .

Assuming the central bank tracks a completely productivity-driven Wicksellian real rate of interest fairly closely (by setting the nominal rate "correctly" for some given endogenous evolution of the (expected) rate of inflation), then the present model is capable of providing an explanation for this puzzle, which is rooted in the initial response of inflation due to the sudden reallocation of the household's liquidity portfolio in favour of real money balances (due the the LM-type demand for real money balances implied by credit production), which has to be accommodated by a sudden drop in actual inflation given the maintained assumption of an underlying "passive" Friedman-type constant money supply growth rate rule, followed by convergence from above (see figure (2)).

Of course, the model at present due to it's parsimonious nature is still very stark in the responses of inflation it produces following a positive shock to productivity, in that the initial drop is of unexpected "one-off" nature, followed by an immediate jump of inflation above steady state and gradual and persistent convergence from above. However, if one were to entertain the view that the central bank actually engaged in "real-time" open-market operations, eliminating "high-frequency" erratic shocks to money demand (and thus inflation) by *inelastically* supplying money so as to partially dampen this, a less stark picture would emerge. Also, as already discussed above, the market for liquidity in the present model is also very parsimonious in structure, as it consists of a simple liquidity portfolio comprising only two means-of-exchange, money and credit and a simple inert evolution of total liquidity demand, modeled by habit persistence in consumption. Contemplating a microfounded view of a more complex and diversified portfolio, and smooth state-contingent substitution between such means-of-exchange, for some given liquidity demand, presumably would lead to a less pronounced variation in the unexpected component of inflation as implied by the model.

4.4 Investment Adjustment Cost & Endogenous Labour

Boldrin et al. (2001) modify a canonical RBC model to include various sources of real rigidities, such as inelastic supply of physical capital, inter-sectoral labour market rigidities and lagged responses of labour to changes in sectoral productivity. More importantly, they also compare their model to the

[13] This analysis of course assumes that the federal funds rate correctly identifies the instrument of monetary policy. Other studies have employed a narrow monetary aggregate, such as M1 (see Sims, 1972; Eichenbaum, 1992)

model setup used in Jermann (1998), which employs the habit-persistence-adjustment cost framework in a similar way as is done in the present paper. The former authors conclude that Jermann's framework *always* leads to countercyclical labour following a one-off shock to productivity. This is indeed true if one decides to calibrate adjustment costs to investment to rise very fast as investment (demand) increases. Figure (8) demonstrates this by plotting the economy's response to a one-off shock to the productivity innovation, but for a much higher calibrated adjustment cost parameter, set to $\kappa = 10.0$ instead of the benchmark calibration of $\kappa = 2.0$. For such an inelastic supply of physical

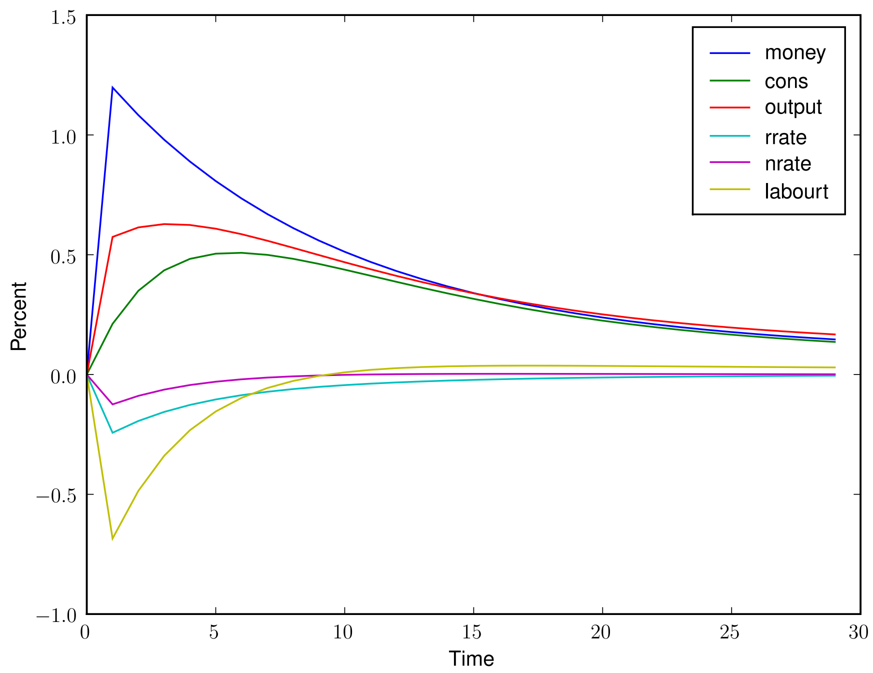

Fig. 8. 1% innovation productivity, $\kappa = 10.0$

capital, following the positive shock to productivity, the saving objective of the representative household is frustrated to an even greater extent, leading to the outcome that the household decides to smooth it's marginal valuation of wealth by taking more leisure and thus working less ("labourt" is the *total* amount of labour). This of course would make the predictions of the model much worse in that labour turns strictly countercyclical - and as a result - would also lead to much smaller volatility of output in general, as the positive shocks to the Solow residual process are dampened by a substitution away from the usual "make hay while the sun shines" effect towards taking more leisure instead. But as I have shown, assuming only moderate amounts of adjustment costs to investment still results in the inverted indicator effect of interest rates, while preserving the procyclical nature of endogenous labour.

5 Conclusion

I have described and solved a monetary real business cycle banking time model, in which self-produced credit (see Kejak and Gillman, 2005; Gillman and Kejak, 2007) with a positive interest rate elasticity *locally* leads to an LM-type demand for real money balances. *Globally*, for different steady state calibrations of the nominal rate of interest, the same money demand function is of Cagan-type (see Cagan, 1956), as higher nominal rates imply a falling interest-rate elasticity (see Gillman and Kejak, 2008). The model thus follows in spirit the cash-in-advance literature but endogenises velocity by introducing a second, self-produced, means-of-exchange which is a perfect complement to money in this sense.

Contrary to common practice in the cash-in-advance literature, I do not model the growth rate of the money supply *stochastically*, but instead assume a Friedman-type constant growth rate rule which is completely deterministic. I justify this decision by showing that the model's *endogenous demand for real money balances* is such as to vary intuitively over the business cycle so as to reproduce closely the co-movement of real balances, consumption and output, and the rate of inflation, which has been documented in many studies (see inter alia Friedman, 1971; King and Watson, 1996).

A key building block required for obtaining this result is to model the supply of physical capital inelastically, by incorporating a quadratic adjustment cost term in the investment process. This has already been shown elsewhere (see Boldrin et al., 2001) to improve the canonical real business cycle framework in as far as it enables an inverted indicator modeling of real interest rates. Intuitively, following a shock to productivity the household wants to smooth it's marginal value of wealth and thus exhibits a strong demand for saving in the periods following the shock. With inelastic capital however, the implementation of this consumption smoothing (saving) objective is quickly frustrated, as capital gains and a "bull market" (increases in Tobin's q) make investing increasingly expensive. The household thus chooses, in some loosely speaking sense, to "overeat" in all periods after the shock relative to tomorrow, leading to a fall in the real rate, as implied by the stochastic discount factor derived from the first-order condition with respect to real bonds.

As the nominal rate - through the Fisher equation - is found to fall as well, the household optimally reallocates it's liquidity supply portfolio in accordance with the LM-type money demand function implied by credit production, favouring real money balances over credit procyclically over the business cycle. Therefore, the household is found to vary the composition of it's portfolio of liquidity supply in response to productivity-driven changes in the nominal rate - which for given inflation expectations and the Fisher equation - derives from the Wicksellian real rate of interest. With habit persistence in consumption, the goods market, in some sense, reacts more sluggishly whereas the financial (liquidity) market reacts instantaneously to interest rate movements. The model's endogenous behaviour off steady state is almost unidirectional from the goods market to the financial market. The only way responses in the financial (liquidity) market feed back into the goods market is through the (expected) inflation tax affecting the marginal rate of substitution between consumption and leisure, which is however small quantitatively (see Cooley and Hansen, 1995).

Following a positive shock to productivity, the sudden reallocation of the household's liquidity portfolio in favour or real money balances - for a given deterministic supply of real balances - requires equilibrium in the market for real money balances to be established through a sudden drop in the inflation rate below it's steady state value, to adjust money supply to be in line with money demand. However, along the subsequent expected projected path back to the steady state, interest rates converge gradually from below, credit is thus gradually favoured again and the demand for real money balances has to fall gradually, residually, while in the meantime because of consumption habit, total liquidity demand is gradually expanding in hump-shaped fashion. Again, for a deterministically given supply of real money balances, the inflation rate has to adjust endogenously so as to equate money demand and supply along the transitional expected path, meaning that inflation after the sudden drop, jumps back up above steady state so as to converge gradually from above.

I have discussed how this behaviour of actual and expected inflation is in some sense capable of giving a partial explanation for the so-called price puzzle observed in impulse responses from identified VARs following a monetary expansionary shock, with the caveat that such studies employ non-unique identification schemes of innovations to structural shocks. The model has been shown to exhibit highly persistent *expected* inflation rate time series in simulations, which are also procyclical. By introducing productivity shocks in credit production which ought to be understood as shocks embodying financial deregulation (see Kejak and Gillman, 2005; Gillman and Kejak, 2007), I have shown how the model is in principle capable of reproducing the breakdown of a stable demand function for real money balances, which is a striking feature of U.S. post-1980s data. Since I have taken a constant deterministic money growth rate rule as my maintained assumption throughout, I believe a natural question one may ask next is whether the current model provides a framework to determine, in some sense, an optimal state-contingent money supply growth rate response, and whether one would want to model this assuming direct control over the monetary aggregate, or only the monetary base, linking the former via the financial accelerator mechanism (see Bernanke et al., 1999) to the latter.

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